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COUPLING THROUGH A WALL BETWEEN TWO FREE CONVECTIVE SYSTEMS

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Abstract—The paper considers the problem of thermal coupling produced by conduction through the wall separating two boundary layers. The features of a fairly general technique capable of handling this problem are then outlined. A particular example of free convection near a vertical plate separating two reservoirs of fluid at different temperatures is considered in detail. The numerical results are then used to discuss the free convection system in particular and the merits of the technique in general.

NOMENCLATURE

X, x,	vertical displacement:
Y, y,	horizontal displacement;
ζ, η,	similarity variables;
U, u,	vertical velocity;
V, v,	horizontal velocity;
Φ, ϕ, T,	temperature;
F, f, ψ,	stream function;
H, G, ξ,	functions defined in text;
W,	plate width;
L,	plate height;
β,	thermal expansion coefficient;
k,	thermal conductivity;
ν, κ,	momentum, thermal diffusivity;
Pr,	Prandtl number;
χ, α,	thermal resistance ratios;
q,	plate heat flux density;
R,	plate thermal resistance per unit area;
Ra,	Rayleigh number $[\beta g(T_{1\infty} - T_{2\infty})L^3/\nu\kappa]$;
Nu,	Nusselt number $[qL/k(T_{1\infty} - T_{2\infty})]$.

Subscripts

$1, 2, i$,	hot, cold, i-th fluid;
o,	$y = \eta = 0$;
∞	$y \to \infty$;
S,	plate.

Superscripts

j,	number of iterations;
$\bar{\ }$,	average;
$'$,	differentiation with respect to η.

INTRODUCTION

CONVECTION on both sides of a thermally-conducting wall or plate is a common occurrence in a wide variety of thermal devices and systems: prominent examples include heat exchange equipment and the convective components of habitable spaces. In a typical situation the heat-transfer problem is broken into a series of sub-problems and handled in either of two ways. If it is satisfactory to lump the conductances of the two convective systems together with that of the intermediate plate, then an approximate overall heat flow analysis may be applied. The objection to this approach is the averaging implicit in the use of conductances which have usually been determined for boundary conditions which do not apply accurately to the circumstances at hand; e.g. they are taken empirically from isothermal surface data. An alternative approach is to first analyse each convective system in detail and obtain an accurate solution for each by replacing the other convective system, along with the intermediate plate, by an "equivalent" boundary condition.

In general, neither of the above approaches can be expected to predict the overall heat-transfer rate accurately and both of them will likely produce considerable error in the local description of the flow fields. This error is related to the inability of both approaches to incorporate a basic feature of the overall system; namely, the coupling produced by conduction through the plate separating the convective systems. This paper discusses a particular technique for accommodating the coupling feature for boundary-layer systems. Although the discussion is restricted to laminar conditions, and the conventional assumptions of negligible viscous dissipation and constant properties, an extension to turbulent boundary layers should present few additional difficulties.

Previous work in this area is extremely scarce and appears to be limited to forced convection systems [1, 2] although a few related conduction-free convection studies have been undertaken [3–5]. From these it is evident that there is a need for further discussion of coupled convective systems in general and of coupled free convective systems in particular. The only broad discussion known to the authors is that of Viskanta and Abrams [2] who have chosen to limit their presentation to results for co-current forced convection systems to which the super-position principle applies. It is very likely that their method would also apply to the corresponding counter-current situation but it is clearly not applicable when the velocity field or the properties are functions of the dependent variables, the temperature in particular. In general, therefore, an alternative method is required and the use of Von Mises' transformation [2] is certainly one possibility, though it remains unproven as yet.

The technique proposed here consists of treating the system in three parts: two boundary layer regions separated by a plate in which the longitudinal conductance is much less than the lateral conductance determining the coupling effect. The boundary layer problems are each treated in a similar way, one as a Dirichlet problem and the other as a Neumann problem, by formulating them for an arbitrary, but unknown, thermal boundary condition on temperature or heat flux. Their solution is facilitated in general through the use of transformations [6–8] of the type:

$$\psi(x, y) = H(\xi) F(\xi, \eta)$$

$$\phi(x, y) = G(\xi) \Phi(\xi, \eta)$$

where ψ and ϕ are the stream function and temperature, respectively, in the physical plane: ξ is a convenient function of x, and η is a "similarity" variable. The functions $H(\xi)$ and $G(\xi)$ are determined from the hydrodynamic and thermal boundary conditions along the plate surface. The use of these transformations in the boundary layer equations reveals that the departure from similarity is related to the ξ-dependency of $F(\xi, \eta)$ and $\Phi(\xi, \eta)$. But since the approximate lumped analyses assume boundary conditions which imply similarity e.g. isothermal plate, it follows that the departure from similarity also indicates the degree of approximation in the lumped analyses. The ability of the transformations to reveal this departure from similarity is thus of central importance.

A characteristic difficulty in finding solutions to coupled convective systems is the absence of freedom to completely prescribe all of the boundary conditions *ab initio*. Instead, some of them can only be introduced by appropriate expressions for continuity and conservation at the surfaces of the plate. In this form the unprescribed boundary conditions automatically couple the convective systems and furthermore they reveal an important set of non-dimensional parameters which include not only the different physical properties of the media involved but also describe the relative importance of each of the three thermal subsystems.

In this paper, attention is focussed on the particular problem of laminar, free convection near a conducting, vertical plate separating two large reservoirs of quiescent fluid, each at a different temperature. Whilst this problem is of interest in itself, it also provides a searching test of the proposed technique for a counter-flow system within which, in addition, the energy equation is not linear in the temperature. Numerical results calculated for air reservoirs will be used to illustrate the departure from similarity produced by the coupling effect. In particular, the plate temperature and heat flux distributions will be discussed.

FORMULATION

The system considered is shown in Fig. 1. High temperature fluid 1 on the right hand side of the plate loses heat through the plate and is thereby cooled, thus producing a descending boundary layer. The ascending boundary layer

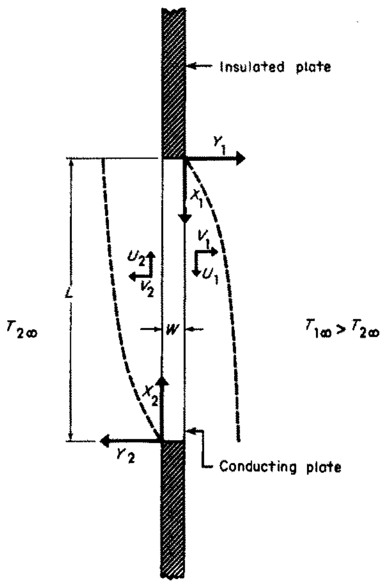

FIG. 1. Physical and coordinate systems.

on the left hand side of the plate is produced by the resultant heating of low temperature fluid 2. In general, the conductivity of the plate would vary from point to point but the specific example chosen here is for a constant conductivity over a length L and zero conductivity elsewhere. The extremities of the conducting zone thus define the leading edges of the counter-current boundary-layer systems.

The boundary layer equations describing each laminar convective system may be written as:

$$\frac{\partial u_i}{\partial x_i} + \frac{\partial v_i}{\partial y_i} = 0$$

$$\frac{1}{Pr_i}\left(u_i \frac{\partial u_i}{\partial x_i} + v_i \frac{\partial u_i}{\partial y_i}\right) = \phi_i + \frac{\partial^2 u_i}{\partial y_i^2} \tag{1}$$

$$u_i \frac{\partial \phi_i}{\partial x_i} + v_i \frac{\partial \phi_i}{\partial y_i} = \frac{\partial^2 \phi_i}{\partial y_i}$$

$$x_i = X_i/L,\ y_i = Ra_i^{\frac{1}{4}} Y_i/L$$

where

$$u_i = \frac{LU_i}{\kappa_i Ra_i^{\frac{1}{4}}},\ v_i = \frac{LV_i}{\kappa_i Ra_i^{\frac{1}{4}}} \text{ and } \phi_i = \frac{|T_i - T_{i\infty}|}{T_{1\infty} - T_{2\infty}}.$$

The corresponding boundary conditions are:

$$y_i = 0:\quad u_i = v_i = 0,\ \phi_i = \phi_{0i}(x)$$
$$[\text{or } \partial\phi_i/\partial y_i = fn(x)]$$
$$y_i = \infty: u_i = \phi_i = 0$$
$$x_i = 0: u_i = \phi_i = 0 \quad [\text{or } \partial\phi_i/\partial y_i = \text{constant}] \tag{2}$$

where $\phi_{0i}(x)$ is the temperature distribution along the i-th side of the plate and is not prescribable. It is found from satisfaction of the heat flux continuity equation given by

$$k_1\left(\frac{\partial T_1}{\partial Y_1}\right)_{Y_1=0} = \frac{k_s}{W}[T_{01}(X_1) - T_{02}(X_2)]$$
$$= -k_2\left(\frac{\partial T_2}{\partial Y_2}\right)_{Y_2=0} \tag{3}$$

with $X_2 = L - X_1$, or, in normalized form,

$$\left[\frac{k_1 Ra_1^{\frac{1}{4}}}{L}\right]\left(\frac{\partial \phi_1}{\partial y_1}\right)_{y_1=0} = \frac{\phi_{01}(x_1) + \phi_{02}(x_2) - 1}{R}$$
$$= \left[\frac{k_2 Ra_2^{\frac{1}{4}}}{L}\right]\left(\frac{\partial \phi_2}{\partial y_2}\right)_{y_2=0}$$

with $x_2 = 1 - x_1$: $R = W/k_s$ is the thermal resistance of the plate per unit area.

It is convenient to transform the governing equations (1) into another set of partial differential equations by putting

$$\psi_i(x_i, y_i) = (\tfrac{4}{3}x_i)^{\frac{3}{4}} f_i(\xi_i, \zeta_i)$$

and

$$\phi_i(x_i, y_i) = \Phi_i(\xi_i, \zeta_i) \qquad (4)$$

where $\psi_i(x_i, y_i)$ is a stream function such that $u_i = \partial\psi_i/\partial y$ and $v_i = -\partial\psi_i/\partial x_i$, with

$$\xi_i = x_i,$$

and $\zeta_i = y_i/(\frac{4}{3}x_i)^{\frac{1}{4}}$ is a similarity variable. Under similarity conditions, f_i and Φ_i are functions of ζ_i alone and therefore, more generally, the solutions may be considered as variations from the similarity solutions which then play the role of base solutions.

Before substituting the above transformations into (1) it is worthwhile considering the limiting forms of the boundary layer equations as $Pr_i \to 0, \infty$. As $Pr_i \to 0$, the viscous terms are known to be least important in the equation of motion whereas as $Pr_i \to \infty$ it is the inertia terms which are least important. In both cases the remaining (i.e. dominant) term of these two is balanced by the only driving term i.e. the buoyancy term. To reflect this physical behavior in the governing equation it is useful [7, 8] to let

$$\eta_i = \left(\frac{Pr_i}{1 + Pr_i}\right)^{\frac{1}{4}} \zeta_i$$

and

$$F_i(\xi_i, \eta_i) = \left(\frac{1 + Pr_i}{Pr_i}\right)^{\frac{1}{4}} f_i(\xi_i, \zeta_i). \qquad (5)$$

It has been noted elsewhere [4] that this Lefevre-type of transformation greatly reduces the importance of the Prandtl number in the resulting problem and thus removes the need to determine separate solutions for a large number of values for Pr_i.

Using the above transformations (4) and (5) in the governing equations then gives:

$$Pr_i F_i''' + F_i F_i'' - \tfrac{2}{3}(F_i')^2 + (1 + Pr_i)\Phi_i = \tfrac{4}{3}\xi_i\left(F_i'\frac{\partial F_i'}{\partial \xi_i} - F_i''\frac{\partial F_i}{\partial \xi_i}\right)$$

$$\Phi_i'' + F_i\Phi_i' = \tfrac{4}{3}\xi_i\left(F_i'\frac{\partial \Phi_i}{\partial \xi_i} - \Phi_i'\frac{\partial F_i}{\partial \xi_i}\right) \qquad (6)$$

where the prime denotes differentiation with respect to η_i. It is immediately apparent from these that when F_i, Φ_i are independent of ξ_i the equations take on a similarity form appropriate to an isothermal plate. Furthermore, it is evident that as $Pr_i \to 0, \infty$ the equations become independent of Prandtl number. Transformation of the boundary conditions (2) leads to:

$$\eta_i = 0: \;\; F_i = F_i' = 0$$
$$\eta_i = \infty: F_i' = \Phi_i = 0. \qquad (7)$$

The missing condition on temperature at $\eta_i = 0$ must be replaced by a pair of conditions,

$$\frac{1}{[\frac{4}{3}\xi_1]^{\frac{1}{4}}}\Phi_1'(\xi_1, 0) = \alpha\frac{1}{[\frac{4}{3}(1 - \xi_1)]^{\frac{1}{4}}}\Phi_2'(1 - \xi_1 0) \qquad (8a)$$

and

$$\Phi_1(\xi_1, 0) + \Phi_2(1 - \xi_1, 0) - 1 = \frac{\chi}{[\frac{4}{3}(1 - \xi_1)]^{\frac{1}{4}}}\Phi_2'(1 - \xi_1, 0) \qquad (8b)$$

where

$$\alpha = \frac{k_2}{k_1}\left(\frac{Ra_2}{Ra_1}\right)^{\frac{1}{4}}\left[\frac{Pr_2(1 + Pr_1)}{Pr_1(1 + Pr_2)}\right]^{\frac{1}{4}}$$

and

$$\chi = \frac{w}{L}\frac{k_2}{k_s} Ra_2^{\frac{1}{4}}\left(\frac{Pr_2}{1 + Pr_2}\right)^{\frac{1}{4}}$$

are the respective thermal resistance ratios for the two boundary layers, and the plate and second boundary layer. Equations (6) together with the boundary and coupling conditions (7) and (8) pose a complete problem.

NUMERICAL SOLUTIONS

In seeking solutions to the transformed boundary-layer equations (6) two broad possibilities appear. Firstly, it is possible to expand the dependent variables in a Görtler, or similar, series which upon substitution generates an infinite set of ordinary differential equations. If the latter can be written in universal forms it is well worthwhile using this method because the problem may then be solved in as general a form as it is possible to describe the temperature and velocity in terms of the original series. But in the present problem the boundary conditions are implicit in the solution as coupling conditions and therefore do not readily lend themselves to such an approach. The second method, which is used here, is to solve the equations (6) as a set of partial differential equations using the concept of local similarity [9], thus reflecting the central role of the similarity solutions. The technique is to solve for F_i and Φ_i numerically for several successive values of ξ_i at each of which any ξ_i-dependence is incorporated explicitly. The starting solution is obtained by simply suppressing the right-hand sides of equations (6). Subsequent integration with respect to η_i is then carried out with the ξ_i-derivatives approximated by explicit finite differences. The method of integration has been discussed and developed elsewhere [7, 8, 10]. It might be mentioned that the grid spacings used were $\Delta\eta_i = 0{\cdot}04$, $\Delta\xi_i = 0{\cdot}05$ with $0 \leqslant \eta_i \leqslant 10$ and $0 \leqslant \xi_i \leqslant 1{\cdot}0$.

The above technique is capable of handling a very wide range of plate temperature distributions providing, of course, that they can be specified in some way. In the present problem this is impossible *ab initio* and therefore the satisfaction of the coupling equations (8) constitutes a matching problem between the individual convective systems. This matching was achieved by a simple iterative process in which one boundary layer system (low temperature) was treated as a Dirichlet problem and the other as a Neumann problem. The Dirichlet solution was initiated by assuming that the zeroth approximation to the plate temperature varied linearly between $T_{1\infty}$ and $T_{2\infty}$ i.e. $\Phi_2^{(0)}(\xi_2, 0) = \xi_2$. The boundary-layer equations (6) were the solved with $i = 2$ to generate the corresponding heat flux, represented by $\Phi_2'(\xi_2, 0)$. The first of equations (8) was used next to determine a zeroth approximation to the heat flux on the other side of the plate. The boundary layer equations were then solved as a Neumann problem with $i = 1$ for all $\xi_1 \neq 0, 1$ since equation (8a) is ill-behaved at these points. This solution generated the zeroth approximation for $\Phi_1(\xi_1, 0)$ which, together with the originally calculated $\Phi_1'(\xi_1, 0)$, was used in the coupling equations (taken together) to determine another set of values for $\Phi_2(\xi_2, 0)$ described as $\hat{\Phi}_2^{(0)}(\xi_2, 0)$, say. Finally, a comparison between $\hat{\Phi}_2^{(0)}(\xi_2, 0)$ and $\Phi_2^{(0)}(\xi_2, 0)$ was necessary to close the iterative loop. In fact, it was found by trial and error that the first approximation $\Phi_2^{(1)}(\xi_2, 0)$ was best taken as the mean of $\Phi_2^{(0)}(\xi_2, 0)$ and $\hat{\Phi}_2^{(0)}(\xi_2, 0)$; or, in general,

$$\Phi_2^{(j+1)}(\xi_2, 0) = \tfrac{1}{2}[\Phi_2^{(j)}(\xi_2, 0) + \hat{\Phi}_2^{(j)}(\xi_2, 0)].$$

The iteration proceeded until $\Phi_2^{(j+1)}(\xi_2, 0)$ and $\Phi_2^{(j)}(\xi_2, 0)$ agreed to within 10^{-3}. Typically, this took 3–6 iterations during which time most of the computational work was performed in the boundary-layer systems. The iteration process is illustrated in Fig. 2a.

One minor point remains to be discussed and this is the involvement of plate data at ξ_1 (or ξ_2) = 0, 1. The difficulty with equation (8a) at these points is readily resolved because
(1) $\Phi_1'(0, 0) = \Phi_2(0, 0) = 0$ identically, and
(2) $\Phi_1(1, 0)$ or $\Phi_2(1, 0)$ may be calculated from equations (8) taken together. That is,

$$\Phi_1(1, 0) - 1 = (\tfrac{3}{4})^{\frac{1}{4}} \frac{\chi}{\alpha} \Phi_1'(1, 0)$$

$$\Phi_2(1, 0) - 1 = (\tfrac{3}{4})^{\frac{1}{4}} \chi \Phi_2'(1, 0) \qquad (9)$$

and consequently the search for two eigenvalues in the integration of equations (6) and (7) is replaced by a simpler search for one eigenvalue when $\xi_i = 1$. But since the profiles at $\xi_i = 1$

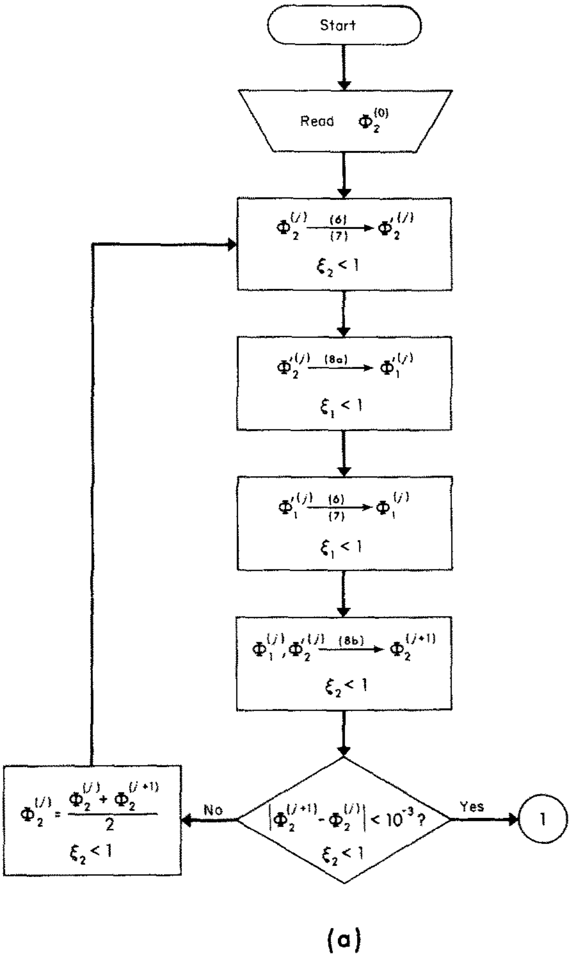

FIG. 2a. Iterative matching scheme.

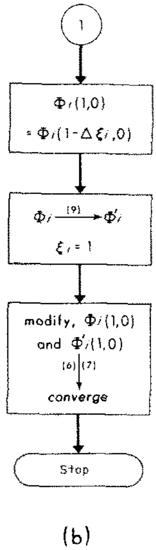

FIG. 2b. Trailing edge scheme.

have no effect on the profiles at $\xi_i < 1$ there is no need to determine them until after matching has been completed elsewhere. This step is shown in Fig. 2b.

RESULTS AND CONCLUSIONS

The particular system chosen for study and presentation here is that in which air is the fluid in both reservoirs. Therefore the results refer to $Pr_1 = Pr_2 = 0{\cdot}72$ and $\alpha = 1$, which leaves one parameter (χ) for discussion. For most plate materials sandwiched between air in laminar flow, $\chi \leqslant 0(1)$. Velocity and temperature profiles plotted for a particular location along the plate revealed virtually no change between $\chi = 0$ and $\chi = 0{\cdot}8$ and therefore it must be concluded that the relative resistance of the plate, as measured by this parameter, has very little effect on the field profiles for air. This conclusion could be less true of profile derivatives.

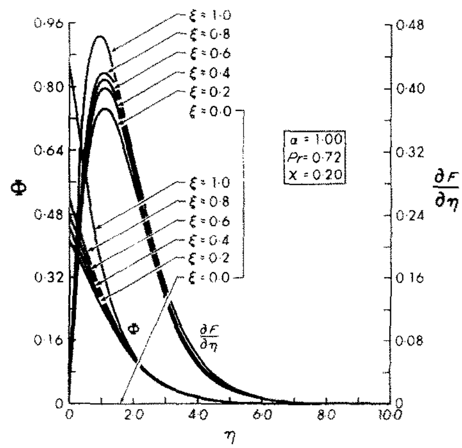

FIG. 3. Velocity and temperature profiles.

Figure 3 shows the field profiles for an arbitrary value of $\chi = 0{\cdot}2$. The curves for each of a series of locations on the plate indicate a substantial ξ-dependency. As expected, this variation is monotone for both velocity and temperature distributions though it is evident that the greatest variations occur near the leading and trailing edges of the conducting zone. The departure from similarity is seen to be significant.

In a lumped conductance analysis of the air–air system the plate would be taken as isothermal. Figure 4 shows the actual plate temperature distribution and reveals a sigmoidal form. The curve for $\chi = 0$ corresponds to a plate offering no thermal resistance between the boundary

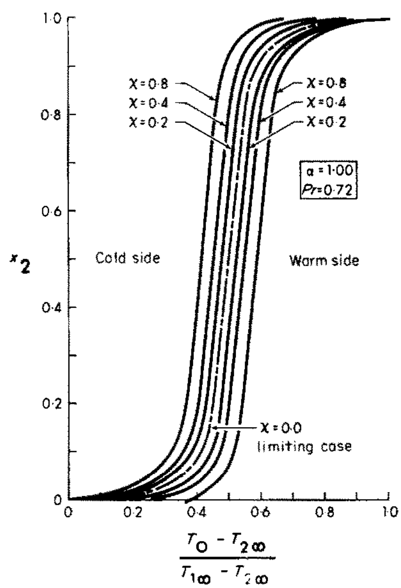

FIG. 4. Plate temperature distribution.

layers. On either side are plotted the high and low temperature surface temperature distributions for each of a set of plate resistances. The curve for $\chi = 0{\cdot}2$, for example, corresponds to a glass window 1·0 cm thick and 0·3 m high when the trailing edge flow is approaching transitional conditions. In agreement with Fig. 3, it is evident that the greatest variations in surface temperature occur within about 20 per cent of the length at each end. Therefore, even if temperature variations over the central region of the plate are neglected it is apparent that departures from an isothermal condition are very substantial over roughly 40 per cent of the plate.

It could be anticipated that the heat flux density would show corresponding departures in the vinicity of the trailing and leading edges. Figure 5 confirms this in a comparison with a

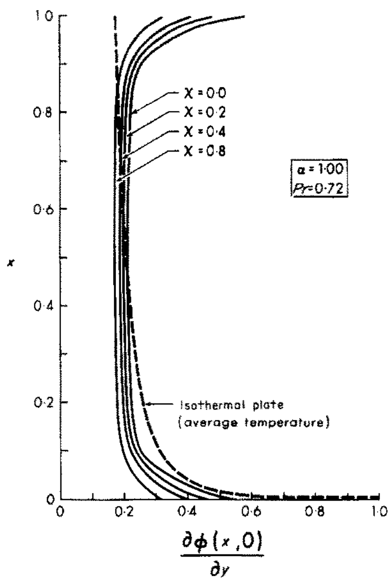

FIG. 5. Plate heat flux density distribution.

curve (for one side*) based on a mean isothermal plate temperature. In essence, the figure illustrates the controlling effect of the leading edge region of each boundary layer, for it is principally in these regions that the local overall conductance is significantly reduced thereby permitting greater heat flux densities and producing a series of curves symmetric about

* The corresponding curve for the other side would exhibit the opposite trend, thus indicating the impossibility of matching with an isothermal surface.

x (or ξ) = 0·5. The effect of plate resistance, for $0 \leqslant \chi \leqslant 0{\cdot}8$, is seen to be moderate.

Integration of the heat flux density plotted in Fig. 5 enables the overall heat flux to be calculated and this is shown non-dimensionally in Fig. 6. For a mean isothermal plate temperature the average Nusselt number is $\overline{Nu} = 0{\cdot}1978\ Ra^{\frac{1}{4}}$ using a lumped analysis. Comparison with the values generated from the present analysis reveals a discrepancy of the order of 10 per cent. The figure also indicates that the Nusselt

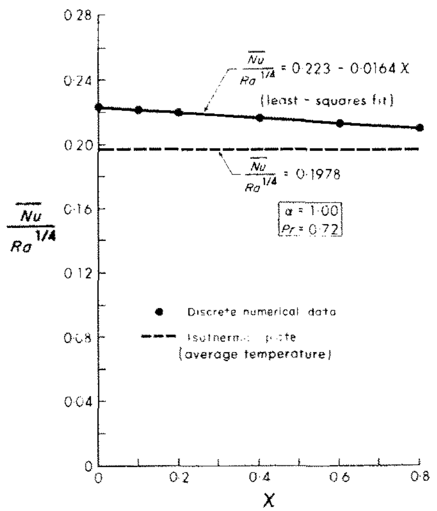

FIG. 6. Overall heat transfer relation.

number reduces with increasing plate resistance, though the effect is seen to be quite small. The expression given suggests that the discrepancy would vanish at $\chi \simeq 1{\cdot}54$ but this is an unusually high value for an air–air system.

The iterative process offers a variety of possibilities for matching and the test of each of these is the rapidity of convergence which they produce. Ideally, perhaps, one might consider a technique such as the Newton–Raphson method but accuracy demands matching at many points along the plate which could lead to an unwieldy matrix. The same comment would apply to similar methods. Furthermore, these methods offer no special advantage unless convergence can only be expected in a narrow range and the number of iterations might otherwise be large. The physical situation considered here places bounds on the solution and even contains information on where the local solution should lie in relation to these bounds. It was therefore no great surprise to find that the Dirichlet–Neumann combination of boundary-layer problems produced a fairly rapid convergence when the iterative loop was simply closed with the mean of two successive values. In fact it was found that the convergence rate did not appear to be too sensitive to the choice of the initial plate temperature distribution, though it did depend on the magnitude of the plate thermal resistance.

This latter dependence may be anticipated by referring to the coupling equations (8) from which it is evident that as χ is increased the thermal separation between the boundary layers increases. Conversely as $\chi \to 0$, the thermal separation vanishes and in this condition the number of iterations required is least. The rapid convergence with $\chi = 0$ was an economical starting point for results in general because it provided a suitably-close approximation with which to enter an iterative loop with $\chi > 0$. In corresponding co-current boundary layer systems the number of iterations would probably be even smaller. For the accuracy stated previously, the computing time was always less than 30 min on the University of Alberta IBM 360/67 machine.

Free convection on both sides of a vertical plate is of intrinsic interest but it also serves as a valuable test of the method in general circumstances when surface temperature variations will be significant and the use of symmetry unlikely. Thus it is reasonable to make broad comments on the suitability and limitations of the method for a much wider range of problems than can actually be given detailed discussion here. In essence the technique will apply to most pairs of boundary layers existing on opposite sides of a long plate, provided that the latter is well approximated by a one-dimensional conducting system. Thus, whilst there appear to be few limitations on the type of boundary layers or fluids which can be treated, the wall separating them must not be thick enough to require a solution to Laplace's equation, nor should it have a conductivity much greater than that of the fluids flowing over it. If the latter condition is not met the wall will tend to offer a thermal short circuit and thus reduce variations in the surface temperature distribution. If the former condition is not met the analysis not invalidated but the coupling equations alone are insufficient to describe the problem accurately and since very little computing time is spent in satisfying the coupling equations it is not likely that the additional solution of Laplace's equation would improve the situation; it simply makes it less economical. There appears to be no reason why the technique could not accommodate moderate variations in local thermal resistance $R(x)$ and therefore it should be applicable to systems involving either multi-material plates or unheated (or insulated) regions.

The stream function and temperature transformations used in this paper offer some restriction in that not all boundary layers may be so described. Furthermore, when the transformations do apply they do not provide the only formulation from which solutions may be developed. However, they do provide a very convenient phrasing of a wide range of boundary-layer problems, especially when fluid properties can be assumed constant.

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COUPLAGE A TRAVERS UNE PAROI ENTRE DEUX SYSTEMES DE CONVECTION NATURELLE

Résumé— L'article traite du problème du couplage thermique par conduction à travers une paroi séparant deux couches limites. On a dégagé les éléments caractéristiques d'une technique générale capable de maîtriser ce problème. Un exemple particulier de convection naturelle le long d'une plaque verticale séparant deux réservoirs de fluide à différentes températures est étudié en détail. Les résultats numériques sont ensuite utilisés pour discuter en particulier le système à convection naturelle et en général les avantages de la technique.

THERMISCHE VERBINDUNG ZWEIER DURCH EINE WAND GETRENNTER SYSTEME MIT FREIER KONVEKTION.

Zusammenfassung—In dieser Arbeit wird das Problem der thermischen Verbindung infolge Wärmeleitung durch eine Wand, die zwei Grenzschichten trennt, untersucht. Die Grundzüge eines ziemlich allgemeinen, für die Behandlung geeigneten Verfahrens werden aufgezeigt. Das spezielle Beispiel der freien Konvektion an einer senkrechten Platte, die zwei Behälter mit Fluiden unterschiedlicher Temperaturen trennt, wird im

Einzelnen betrachtet. Die numerischen Ergebnisse werden dann verwendet um das System mit freier Konvektion im Besonderen und den Vorteil des aufgezeigten Lösungs-Verfahrens im Allgemeinen zu besprechen.

ТЕПЛОВОЙ КОНТАКТ ДВУХ КОНВЕКТИВНЫХ СИСТЕМ ПУТЕМ ТЕПЛОПРОВОДНОСТИ ЧЕРЕЗ СТЕНКУ

Аннотация—В статье рассматривается проблема теплового контакта путем теплопроводности через стенку, разделяющую два слоя. Приводится схематическое описание довольно обшей методики, применимой для решения этой задачи. Подробно рассматривается частный случай свободной конвекции вблизи вертикальной пластины, разделяющей два сосуда с жидкостью при различной температуре. Численные результаты используются затем для частного случая системы со свободной конвекцией, а тажке для обсуждения общих достоинств этого метода.